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The effect of singlet break-up states in deuteron stripping reactions

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Abstract. The contribution from deuteron break up into spin-singlet, zero relative angular momentum, continuum states, to the transition matrix for (p, d) and (d, p) reactions is examined. It is shown how these effects can be incorporated naturally into an adiabatic approach and results of numerical calculations are presented for some specific reactions. It is found that, within limits set by current knowledge of the isovector spin–orbit component of the nucleon optical potential, the corrections to a theory which neglects singlet break-up states are in general very small.

1. Introduction

The adiabatic theory of deuteron stripping reactions associated with transitions between bound nuclear states (Johnson and Soper 1972) has been shown to give a good account of (d, p) and (p, d) differential cross sections without the use of a lower radial cut-off, and without the use of a purely phenomenological deuteron optical potential for a wide range of targets and deuteron energies in the range 20–55 MeV (Harvey and Johnson 1971, McAllen *et al* 1971, Satchler 1971, Preedom 1972, Blum *et al* 1973, Cooper *et al* 1974, Nolen *et al* 1974, Pignanelli *et al* 1973). In the work of Johnson and Soper (1972) the effects of coupling between the elastic deuteron channel and spin triplet relative S-wave continuum states of the neutron–proton system are included in an approximate way‡. The restriction of the adiabatic theory to S-wave states is consistent with the other approximations used in the theory, but the restriction to triplet states has been supported only by the knowledge that under certain reasonable conditions (no iso-spin dependence of the nucleon spin–orbit force), the singlet states cannot contribute to the reaction.

In this paper the effects of the singlet S-wave channel are quantitatively assessed in some specific cases. The necessary formalism is developed in § 2 and the results are presented in § 3. A final discussion is included in § 4.

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‡ This method has recently been extended (Johnson 1973 unpublished) to include the effects of tensor force components in the neutron–proton force (Johnson and Santos 1971), but these effects will not be considered here.

2. Formalism

2.1. The stripping matrix element

Ignoring explicit contributions involving excited states of the target and residual nuclei, the stripping amplitude can be expressed in a familiar way (Austern 1970) in terms of the quantity

$$T(\mathrm{d},\mathrm{p}) = (\psi^{(-)}_{\boldsymbol{k}_2\sigma_2}(\mathrm{p})\psi^{\xi}_{jl}(\mathrm{n})|\,V_{\mathrm{np}}|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}(\mathrm{p},\mathrm{n})),$$

$$= \int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{R}\;\psi^{(-)*}_{\boldsymbol{k}_2\sigma_2}(\gamma\boldsymbol{R}+\tfrac{1}{2}(\gamma-2)\boldsymbol{r})\psi^{\xi*}_{jl}(\boldsymbol{R}+\tfrac{1}{2}\boldsymbol{r})V_{\mathrm{np}}(r)\psi_{\boldsymbol{k}_1\sigma_1}(\boldsymbol{R},\boldsymbol{r}). \tag{1}$$

where $\gamma = M_A/(M_\mathrm{n}+M_A) \simeq A/(A+1)$ where A is the atomic number of the target nucleus. In equation (1), $\psi^{(-)}_{\boldsymbol{k}_2\sigma_2}(\mathrm{p})$ is the optical model distorted wavefunction for the proton, $\psi^{\xi}_{jl}(\mathrm{n})$ is the form factor for the reaction (Philpott *et al* 1968), and V_{np} is the neutron–proton interaction potential. The function $\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}(\mathrm{p},\mathrm{n})$ is the projection onto the ground state of the target, of the many-body wavefunctions describing the deuteron–nucleus system, having asymptotically a plane wave of deuterons in the incident channel.

Using the techniques which have been used in the approximate treatment of finite range effects (Bencze and Zimanyi 1964, Buttle and Goldfarb 1964), the matrix element (1) (abbreviated by $(\sigma_2\xi\sigma_1)$) can be written

$$(\sigma_2\xi\sigma_1) = \int \mathrm{d}\boldsymbol{R}\;\psi^{(-)*}_{\boldsymbol{k}_2\sigma_2}(\gamma\boldsymbol{R})\psi^{\xi*}_{jl}(\boldsymbol{R})(\boldsymbol{R},\boldsymbol{K}|\,V_{\mathrm{np}}|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}) \tag{2}$$

where the last factor in equation (2) is defined as

$$(\boldsymbol{R},\boldsymbol{K}|\,V_{\mathrm{np}}|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}) = \int \mathrm{d}\boldsymbol{r}\;\mathrm{e}^{-\mathrm{i}\boldsymbol{K}.\boldsymbol{r}}V_{\mathrm{np}}(r)\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}(\boldsymbol{R},\boldsymbol{r}). \tag{3}$$

The operator $\boldsymbol{K}$ is a combination of operators acting only on the neutron and proton wavefunctions, defined by

$$\boldsymbol{K} = \tfrac{1}{2}[(2-\gamma)\boldsymbol{K}_\mathrm{p} - \boldsymbol{K}_\mathrm{n}] = \tfrac{1}{2}\mathrm{i}(\nabla_\mathrm{n} + (\gamma-2)\nabla_\mathrm{p}). \tag{4}$$

In the zero-range approximation (Bencze and Zimanyi 1964, Buttle and Goldfarb 1964), equation (3) is replaced by

$$(\sigma_2\xi\sigma_1) = \int \mathrm{d}\boldsymbol{R}\psi^{(-)*}_{\boldsymbol{k}_2\sigma_2}(\gamma\boldsymbol{R})\psi^{\xi*}_{jl}(R)(\boldsymbol{R},\boldsymbol{K}=0|\,V_{\mathrm{np}}|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}). \tag{5}$$

The equation satisfied by the function $\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}$ can be written

$$(E - T_R - H_{\mathrm{np}} - V_\mathrm{e}(\mathrm{n},\mathrm{p}))|\psi^{(-)}_{\boldsymbol{k}_1\sigma_1}) = 0, \tag{6}$$

where asymptotically,

$$(\boldsymbol{R},\boldsymbol{r}|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}) \underset{\boldsymbol{R}\to\infty}{\sim} \mathrm{e}^{\mathrm{i}\boldsymbol{k}_1.\boldsymbol{R}}\chi^{\sigma_1}_1(\mathrm{p},\mathrm{n})\phi_\mathrm{d}(r) + (\text{outgoing waves}) \tag{7}$$

and

$$H_{\mathrm{np}} = T_\mathrm{r} + V_{\mathrm{np}} \tag{8}$$

$$(H_{\mathrm{np}} + \epsilon_\mathrm{d})\phi_\mathrm{d}(r) = 0; \tag{9}$$

T_R and T_r are kinetic energy operators, ϵ_d is the deuteron binding energy, E is the total energy of the system, and $\phi_d(r)$ is the deuteron bound state wavefunction (assumed here to be a pure ^3S state). The spin function is defined as

$$\chi^{\sigma_1}_{s_1}(\mathrm{p,n}) = \sum_{\sigma_2\sigma_3} (\tfrac{1}{2}\sigma_2 \tfrac{1}{2}\sigma_3 | s_1\sigma_1)\chi^{\sigma_2}_{1/2}(\mathrm{p})\chi^{\sigma_3}_{1/2}(\mathrm{n}), \tag{10}$$

where $\chi^{\sigma_2}_{1/2}(\mathrm{p})$ and $\chi^{\sigma_3}_{1/2}(\mathrm{n})$ are spinors describing the state of the proton and neutron respectively. The effective interaction $V_e(\mathrm{n,p})$ is assumed to have the form (Johnson and Soper 1970)

$$V_e(\mathrm{n,p}) = V_n(\boldsymbol{R}+\tfrac{1}{2}\boldsymbol{r}) + V_p(\boldsymbol{R}-\tfrac{1}{2}\boldsymbol{r}) + V(\boldsymbol{R}), \tag{11}$$

where V_n and V_p are optical potentials evaluated at half the incident deuteron energy, and $V_c(\boldsymbol{R})$ is the Coulomb potential, which in this model is assumed to act on the centre of mass of the deuteron.

Spin projection operators P_{s_1} for the neutron–proton system are now defined, where P_0 is an operator projecting out spin-zero (singlet) wavefunctions, and P_1 is an operator projecting out spin-one (triplet) wavefunctions, and

$$[P_{s_1}, H_{np}] = 0, \qquad P_0 + P_1 = 1. \tag{12}$$

The zero-range matrix element can now be written,

$$(\sigma_2\xi\sigma_1) = \sum_{s_1}\int \mathrm{d}\boldsymbol{R}\,(\boldsymbol{R}, \boldsymbol{K}=0|V^{s_1}_{np}P_{s_1}|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1})\psi^{(-)*}_{\boldsymbol{k}_2\sigma_2}(\gamma R)\psi^{\xi *}_{jl}(\boldsymbol{R}), \tag{13}$$

where V^0_{np} and V^1_{np} are the neutron–proton potentials in the singlet and triplet spin states respectively.

Defining

$$|V_{s_1}\rangle = V^{s_1}_{np}|\boldsymbol{K}=0\rangle, \tag{14}$$

and

$$\psi^{-s_1}_{\boldsymbol{k}_1\sigma_1}(\boldsymbol{R}) = (\boldsymbol{R}V_{s_1}|P_{s_1}|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1})/\langle V_{s_1}|\phi_d\rangle, \tag{15}$$

the matrix element (13) becomes

$$(\sigma_2\xi\sigma_1) = \int \mathrm{d}\boldsymbol{R}\,\psi^{(-)*}_{\boldsymbol{k}_2\sigma_2}(\gamma R)\psi^{\xi *}_{jl}(\boldsymbol{R})\langle V_1|\phi_d\rangle\left(\bar{\psi}^1_{\boldsymbol{k}_1\sigma_1}(\boldsymbol{R}) + \frac{\langle V_0|\phi_d\rangle}{\langle V_1|\phi_d\rangle}\bar{\psi}^0_{\boldsymbol{k}_1\sigma_1}(\boldsymbol{R})\right). \tag{16}$$

Inner products in the space of $\boldsymbol{r}$ only, are denoted by angular brackets, while those in the full space are denoted by round brackets.

The quantity $\langle V_1|\phi_d\rangle$ is the zero momentum Fourier transform of the product $V_{np}(r)\phi_d(r)$, and is the usual constant which appears in the zero-range DWBA limit, eg the constant D_0 of Johnson and Santos (1971)†.

The equations for $\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}$ and $\bar{\psi}^{s_1}_{\boldsymbol{k}_1\sigma_1}$ (equations (6), (14) and (15)), lead to the following coupled equations

$$(E-\epsilon_1-T_R)\bar{\psi}^1_{\boldsymbol{k}_1\sigma_1} = \langle V_1|P_1(H_{np}-\epsilon_1+V_e)|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1})/\langle V_1|\phi_d\rangle, \tag{17a}$$

$$(E-\epsilon_0-T_R)\bar{\psi}^0_{\boldsymbol{k}_1\sigma_1} = \langle V_0|P_0(H_{np}-\epsilon_0+V_e)|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1})/\langle V_0|\phi_d\rangle, \tag{17b}$$

where ϵ_0 and ϵ_1 are at present arbitrary, and the boundary conditions are those appropriate to a beam of (triplet) deuterons in the incident channel. It is at this point that the

† Expressions similar to equation (16) for the case of a rank-one separable triplet V_{np} have been given by Bouldin and Levin (1971, 1972).

adiabatic approximation is introduced. Noting that the function $\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}(\mathrm{p,n})$ is needed only within the range of the neutron–proton interaction, the triplet component is approximated by

$$(\boldsymbol{R}, \boldsymbol{r}|P_1|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}) \simeq \phi_1(r)\tilde{\psi}^1_{\boldsymbol{k}_1\sigma_1}(\boldsymbol{R}), \tag{18}$$

for $|\boldsymbol{r}|$ less than the range of $V^1_{\mathrm{np}}(r)$. This approximation is equivalent to the approximation used in Johnson and Soper (1970) when $\phi_1(r)$ is identified with the deuteron wavefunction. This approximation may be arrived at by assuming that the dominant components of $V_{\mathrm{np}}P_1|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}\rangle$ correspond to relative S-wave eigenstates of H_{np}, with energies small compared with the depth of V_{np}. In this case, it is to be expected that the shape of all the important neutron–proton eigenfunctions will be similar for $|\boldsymbol{r}|$ less than the range of $V_{\mathrm{np}}(r)$, and since $\phi_{\mathrm{d}}(r)$ is the dominant component of $|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}\rangle$, $\phi_{\mathrm{d}}(r)$ should be a good approximation to $\phi_1(r)$. Clearly, if this approximation is reliable, other low energy eigenstates of H_{np} could also be used for $\phi_1(r)$, which would lead to different contributions from the H_{np} terms on the right-hand side of equations (17). The adiabatic approximation assumes that the uncertainty associated with these different choices is not important, and therefore that the important components of $\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}$ involve eigenstates of H_{np} with energies small compared with the other energies associated with the system. The choice

$$\phi_1(r) = \phi_{\mathrm{d}}(r), \qquad \epsilon_1 = -\epsilon_{\mathrm{d}}, \tag{19}$$

has the obvious advantage of ensuring that the dominant elastic scattering component of $P_1|\psi^{(+)}_{\boldsymbol{k}\sigma_1}\rangle$ has the correct asymptotic momentum. In this case, for consistency, $\tilde{\psi}^1_{\boldsymbol{k}_1\sigma_1}$ must be identified with $\bar{\psi}^1_{\boldsymbol{k}_1\sigma_1}$. Using equations (18) and (19) and analogous approximations for the singlet channel leads to

$$(\boldsymbol{R}, \boldsymbol{r}|P_1|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}) \simeq \phi_{\mathrm{d}}(r)\bar{\psi}^1_{\boldsymbol{k}_1\sigma_1}(\boldsymbol{R}), \tag{20a}$$

with $|\boldsymbol{r}|$ less than the range of $V^1_{\mathrm{np}}(r)$, and

$$(\boldsymbol{R}, \boldsymbol{r}|P_0|\psi^{(+)}_{\boldsymbol{k}_1\sigma_1}) \simeq \phi_0(\boldsymbol{r})\frac{\langle V_0|\phi_{\mathrm{d}}\rangle}{\langle V_0|\phi_0\rangle}\bar{\psi}^0_{\boldsymbol{k}_1\sigma_1}(\boldsymbol{R}), \tag{20b}$$

for $|\boldsymbol{r}|$ less than the range of $V^0_{\mathrm{np}}(r)$, where $\phi_0(r)$ is some low energy singlet eigenstate of H_{np} with energy ϵ_0. It is natural to identify the state $\phi_0(\boldsymbol{r})$ with the singlet scattering state at the 'resonant' energy ($\sim 0{\cdot}06$ MeV), but within the framework of the adiabatic approximation, there is no real reason for doing this, and it is to be expected that any other low energy scattering state would be equally suitable. Just as the bound state deuteron energy was chosen for the triplet channel, however, on the grounds that the dominant part of the wavefunction will then have the correct asymptotic momentum, it can be argued that the dominant singlet break up will be to the 'resonant' state (Kolltveit and Nagatani 1969) and choosing the 'resonant' energy for ϵ_0, gives the correct asymptotic momentum for this component of the singlet wavefunction. The presence of the 'resonance' affects mainly the amplitude of the singlet neutron–proton wavefunction in this energy region, and not the shape for $|\boldsymbol{r}|$ less than the range of V^0_{np}. From the point of view of the adiabatic approximation, it is the shape of the wavefunction which is most important, and the approximations (20) rely on the shape of all the important low energy eigenfunctions of H_{np} being similar within the range of the neutron–proton force.

Assuming that the range of $V_{np}(r)$ is the same in both the triplet and singlet states, and that the potentials differ mainly in strength, substitution of equations (20) into equations (17) leads to

$$(E_d - T_R - \bar{V}_{11}(R))\bar{\psi}^1_{\boldsymbol{k}_1\sigma_1} = \left(\frac{\alpha}{\alpha'}\right)\bar{V}_{10}(R)\bar{\psi}^0_{\boldsymbol{k}_1\sigma_1}, \tag{21a}$$

$$(E_d - \epsilon_d - \epsilon_0 - T_R - \bar{V}_{00}(R))\bar{\psi}^0_{\boldsymbol{k}_1\sigma_1} = \bar{V}_{01}(R)\bar{\psi}^1_{\boldsymbol{k}_1\sigma_1} \tag{21b}$$

where

$$\bar{V}_{ij} = \frac{\langle V_i|P_iV_eP_j|\phi_j\rangle}{\langle V_i|\phi_j\rangle}, \qquad (i,j = 0,1\,;\,\phi_1 = \phi_d) \tag{22}$$

and

$$\alpha = \frac{\langle V_0|\phi_d\rangle}{\langle V_1|\phi_d\rangle}, \qquad \alpha' = \frac{\langle V_0|\phi_0\rangle}{\langle V_1|\phi_0\rangle}, \tag{23}$$

and

$$E_d = E + \epsilon_d. \tag{24}$$

In the zero-range approximation, these expressions are very simple, for example

$$\bar{V}^{ZR}_{11}(R) = (P_{1s}V_e(\boldsymbol{r},\boldsymbol{R})P_{1S})_{r=0}, \tag{25}$$

with similar equations for $\bar{V}_{00}$, $\bar{V}_{10}$ and $\bar{V}_{01}$. P_{1S} is a product of spin triplet and relative S-wave projection operators. The boundary condition of an incident plane wave of triplet deuterons defines the following asymptotic forms for the solutions

$$\bar{\psi}^1_{\boldsymbol{k}_1\sigma_1}(\boldsymbol{R}) \underset{\boldsymbol{R}\to\infty}{\sim} \chi_1^{\sigma_1}\,e^{i\boldsymbol{k}_1\cdot\boldsymbol{R}} + (\text{outgoing waves}), \tag{26a}$$

$$\bar{\psi}^0_{\boldsymbol{k}_1\sigma_1}(\boldsymbol{R}) \underset{\boldsymbol{R}\to\infty}{\sim} \chi_0\,(\text{outgoing waves}), \tag{26b}$$

with the usual modifications for Coulomb effects.

Since it has been assumed, in the adiabatic approach, that the dependence of the equations upon the energies ϵ_0 and ϵ_d is very weak, the only way in which the detailed properties of the singlet neutron–proton system enter the calculation is through the factors α and α'. In fact, since the ranges of the $V^{s_1}_{np}$ are assumed to be equal, α and α' are equal to the ratio of the strengths of V^0_{np} and V^1_{np},

$$\alpha = \alpha' = V^0_{np}/V^1_{np}. \tag{27}$$

The direct singlet contribution to the stripping matrix element in (16) thus differs from the triplet contribution by the factor α, but it is clear from the coupled equations, that even when α tends to zero, the singlet still affects the triplet wavefunctions, and hence the triplet matrix element, in second order.

2.2. Partial wave expansion

Making partial wave expansions of the functions $\bar{\psi}^{s_1}_{\boldsymbol{k}_1\sigma_1}$ leads to the following equations

$$\bar{\psi}^{s_1}_{\boldsymbol{k}_1\sigma_1}(\boldsymbol{R}) = \sum_{\sigma_1'} \psi^{s_1}_{\boldsymbol{k}_1\sigma_1\sigma_1'}(\boldsymbol{R})\chi_1^{\sigma_1'}(\text{p, n}), \tag{28}$$

$$\bar{\psi}^{s_1}_{\boldsymbol{k}_1\sigma_1\sigma_1'}(\boldsymbol{R}) = 4\pi \sum_{\substack{l_1\lambda_1\lambda_1' \\ j_1\xi_1'}} i^{l_1}(l_1\lambda_1 1\sigma_1|j_1\xi_1)(l_1\lambda_1' s_1\sigma_1'|j_1\xi_1)\times Y^{\lambda_1}_{l_1}{}^*(\hat{\boldsymbol{k}}_1)Y^{\lambda_1'}_{l_1}(\hat{\boldsymbol{R}})f^{s_1}_{l_1j_1}(k_1R). \tag{29}$$

The radial functions have the following normalization

$$f^{1}_{l_1 j_1}(k_1 R) \underset{R\to\infty}{\sim} j_{l_1}(k_1 R) + (\text{outgoing waves}), \tag{30a}$$

$$f^{0}_{l_1 l_1}(k_1 R) \underset{R\to\infty}{\sim} (\text{outgoing waves}), \tag{31b}$$

and satisfy the following coupled equations

$$[E_d - T_{l_1} - \overline{V}^{l_1 j_1}_{11}(R)] f^{1}_{l_1 j_1}(R) = \delta_{l_1 j_1} \Delta V^{l_1}_{10}(R) f^{0}_{l_1 l_1}(R), \tag{32a}$$

$$[E_d - \epsilon_d - \epsilon_0 - T_{l_1} - \overline{V}^{l_1}_{00}(R)] f^{0}_{l_1 l_1}(R) = \Delta V^{l_1}_{01}(R) f^{1}_{l_1 l_1}(R) \tag{32b}$$

where

$$V^{l_1 j_1}_{11}(R) = \langle (l_1 1) j_1 \xi_1 V_1 | V_e | (l_1 1) j_1 \xi_1, \phi_d \rangle / \langle V_1 | \phi_d \rangle, \tag{33a}$$

$$\overline{V}^{l_1}_{00}(R) = \langle (l_1 0) l_1 \lambda_1 V_0 | V_e | (l_1 0) l_1 \lambda_1, \phi_0 \rangle / \langle V_0 | \phi_0 \rangle, \tag{33b}$$

$$\Delta \overline{V}^{l_1}_{10}(R) = (\alpha/\alpha') \langle (l_1 1) l_1 \xi_1 V_1 | V_e | (l_1 0) l_1 \lambda_1, \phi_0 \rangle / \langle V_0 | \phi_0 \rangle, \tag{33c}$$

$$\Delta \overline{V}^{l_1}_{01}(R) = \langle (l_1 0) l_1 \lambda_1 V_0 | V_e | (l_1 1) l_1 \xi_1, \phi_d \rangle / \langle V_0 | \phi_d \rangle, \tag{33d}$$

and T_{l_1} is the radial kinetic energy operator

$$T_{l_1} = -\frac{\hbar^2}{2\mu_1}\left(\frac{d^2}{dR^2} + \frac{2}{R}\frac{d}{dR} - \frac{l_1(l_1+1)}{R^2}\right). \tag{34}$$

Assuming now that the neutron and proton spin–orbit potentials differ only in their strength, and have the same radial shape of the usual derivative form, the potentials appearing in equations (33) may be evaluated using the following results

$$\begin{aligned} P_S[V_p(r_p)\boldsymbol{l}_p \cdot \boldsymbol{\sigma}_p + V_n(r_n)\boldsymbol{l}_n \cdot \boldsymbol{\sigma}_n] P_S \\ = \frac{1}{R}\frac{d}{dR} P_S[\tfrac{1}{2}(V^{so}_p + V^{so}_n)\tfrac{1}{2}(\boldsymbol{\sigma}_n + \boldsymbol{\sigma}_p) \cdot \boldsymbol{L} f(|\boldsymbol{R} + \tfrac{1}{2}\boldsymbol{r}|) \\ + \tfrac{1}{2}(V^{so}_p - V^{so}_n)\tfrac{1}{2}(\boldsymbol{\sigma}_n - \boldsymbol{\sigma}_p) \cdot \boldsymbol{L} f(|\boldsymbol{R} - \tfrac{1}{2}\boldsymbol{r}|)] P_S \end{aligned} \tag{35}$$

where

$$V_p(r_p) = \frac{V^{so}_p}{r_p}\frac{d}{dr_p} f(|\boldsymbol{r}_p|), \qquad V_n(r_n) = \frac{V^{so}_n}{r_n}\frac{d}{dr_n} f(|\boldsymbol{r}_n|), \tag{36a}$$

$$P_S = \text{n–p relative S-wave propagator}, \tag{36b}$$

$$\boldsymbol{L} = i^{-1}\boldsymbol{R} \times \nabla_R. \tag{36c}$$

Also,

$$\langle (ls) j\xi | \tfrac{1}{2}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \boldsymbol{L} | (l's') j'\xi' \rangle = \delta_{\xi\xi'}\delta_{ll'}\delta_{jj'}\delta_{ss'}\tfrac{1}{2}[j(j+1) - l(l+1) - s(s+1)], \tag{37}$$

$$\begin{aligned} &\langle (l1) j\xi | \tfrac{1}{2}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \boldsymbol{L} | (l'0) j'\xi' \rangle \\ &\quad = \langle (l0) j\xi | \tfrac{1}{2}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \boldsymbol{L} | (l'1) j'\xi' \rangle, \\ &\quad = \delta_{\xi\xi'}\delta_{jj'}\delta_{ll'}\delta_{lj}[l(l+1)]^{1/2}. \end{aligned} \tag{38}$$

The potentials (33) reduce to

$$\begin{aligned} \overline{V}^{l_1 j_1}_{11}(R) = \overline{V}^{1}_{\text{real}}(R) + i\overline{W}^{1}_{\text{imag}}(R) + \tfrac{1}{2}(V^{so}_n + V^{so}_p)\frac{1}{R}\frac{d}{dR}\frac{\langle V_1 | f(\boldsymbol{r}, \boldsymbol{R}) | \phi_d \rangle}{\langle V_1 | \phi_d \rangle} \\ \times \tfrac{1}{2}[j(j+1) - l(l+1) - 2], \end{aligned} \tag{39a}$$

$$\overline{V}^{l_1}_{00}(R) = \overline{V}^{0}_{\text{real}}(R) + \mathrm{i}\overline{W}^{0}_{\text{imag}}(R), \tag{39b}$$

$$\Delta V^{l}_{10}(R) = \frac{\langle V_0|\phi_d\rangle}{\langle V_1|\phi_d\rangle}\tfrac{1}{2}(V^{\text{so}}_{\text{p}} - V^{\text{so}}_{\text{n}})\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}R}\frac{\langle V_1|f(\boldsymbol{r},\boldsymbol{R})|\phi_0\rangle}{\langle V_0|\phi_0\rangle}[l(l+1)]^{1/2} \tag{39c}$$

$$\Delta V^{l}_{01}(R) = \tfrac{1}{2}(V^{\text{so}}_{\text{p}} - V^{\text{so}}_{\text{n}})\frac{1}{R}\frac{\mathrm{d}}{\mathrm{d}R}\frac{\langle V_0|f(\boldsymbol{r},\boldsymbol{R})|\phi_d\rangle}{\langle V_0|\phi_d\rangle}[l(l+1)]^{1/2} \tag{39d}$$

where

$$\langle V_i|\phi_i\rangle\overline{V}^{l}_{\text{real}}(R) = V^{0}_{\text{p}}\langle V_i|f_{\text{pr}}(\boldsymbol{R}-\tfrac{1}{2}\boldsymbol{r})|\phi_i\rangle + V^{0}_{\text{n}}\langle V_i|f_{\text{nr}}(\boldsymbol{R}+\tfrac{1}{2}\boldsymbol{r})|\phi_i\rangle, \tag{40}$$

where f_{pr} and f_{nr} are the form factors for the proton and neutron real potentials respectively, with a similar formula for $\overline{W}^{l}_{\text{imag}}$. It is now clear, that if the neutron and proton spin–orbit potentials are equal, then the stripping is determined purely by the triplet states, since the coupling term vanishes, and no transitions between different total spin states are possible. Rewriting the stripping matrix element (16) as

$$(\sigma_2\xi\sigma_1) = \langle V_1|\phi_d\rangle\{(\sigma_2\xi\sigma_1)^1 + \alpha(\sigma_2\xi\sigma_1)^0\}, \tag{41}$$

where

$$(\sigma_2\xi\sigma_1)^{s_1} = \int \mathrm{d}\boldsymbol{R}\,\psi^{(-)*}_{k_2\sigma_2}(\gamma\boldsymbol{R})\psi^{\xi *}_{jl}(\boldsymbol{R})\overline{\psi}^{s_1}_{k_1\sigma_1}(\boldsymbol{R}), \tag{42}$$

and inserting the partial wave expansions of $\overline{\psi}^{s_1}_{k_1\sigma_1}(\boldsymbol{R})$ and the proton wavefunction, and expanding the form factor as

$$\psi^{\xi}_{jl}(\boldsymbol{R}) = \sum_{\lambda\sigma_3'}(l\lambda s_3\sigma_3'|j\xi)\,\mathrm{i}^l Y^{\lambda}_{l} R_{jl}\chi^{\sigma_3'}_{s_3}, \tag{43}$$

leads to

$$(\sigma_2\xi\sigma_1)^{s_1} = (4\pi)^{3/2}\hat{s}_1\sum_{l_1J_1l_2J_2}\hat{l}_1\hat{l}_2\,\mathrm{i}^{l_1-l_2-l}H_{J_1l_1\sigma_1J_2l_2\sigma_2j\xi}(\hat{\boldsymbol{k}}_1,\hat{\boldsymbol{k}}_2)$$
$$\times\begin{Bmatrix} l_1 & s_1 & J_1 \\ l_2 & \frac{1}{2} & J_2 \\ l & \frac{1}{2} & j \end{Bmatrix}(l_1 0 l_2 0|l0)F^{s_1}_{J_1l_1J_2l_2l}, \tag{44}$$

where

$$H_{J_1l_1\sigma_1J_2l_2\sigma_2j\xi}(\hat{\boldsymbol{k}}_1,\hat{\boldsymbol{k}}_2)$$
$$= \hat{J}_1\hat{J}_2\sum_{m_1M_1m_2M_2}(J_1M_1J_2M_2|j\xi)(l_1m_1 1\sigma_1|J_1M_1)$$
$$\times(-1)^{1/2-\sigma_2}(l_2m_2\tfrac{1}{2}-\sigma_2|J_2M_2)Y^{m_1*}_{l_1}(\hat{\boldsymbol{k}}_1)Y^{m_2*}_{l_2}(\hat{\boldsymbol{k}}_2) \tag{45}$$

and

$$F^{s_1}_{J_1l_1J_2l_2l} = \int_0^\infty f^{s_1}_{l_1J_1}(R)f_{l_2J_2}(\gamma R)R_{jl}(R)R^2\,\mathrm{d}R. \tag{46}$$

The factor in curly brackets in equation (44) is a $9J$ symbol (Brink and Satchler 1962).

2.3. General considerations

Although the formulae above give the stripping matrix element in the zero-range approximation only, finite range effects can be incorporated readily by using the approximate techniques used in the conventional theory (Johnson and Soper 1970,

Bencze and Zimanyi 1964, Buttle and Goldfarb 1964, Harvey 1970). Neglecting for the moment the coupling to the singlet channel, the calculation of the stripping amplitude using the adiabatic theory involves the use of the adiabatic potential $\bar{V}^{11}(R)$ instead of the usual deuteron optical-model potential. It must be emphasized again (Johnson and Soper 1970), however, that the adiabatic potential is not expected to yield the observed elastic scattering, since the wavefunction which it generates, contains for any finite $|\boldsymbol{R}|$ components corresponding to the motion of broken-up neutron–proton pairs. The results of the use of the adiabatic potential instead of the conventional deuteron optical potential have been discussed elsewhere. In a previous paper (Harvey and Johnson 1971) it was shown that the results of the two theories can differ significantly, and that the use of the adiabatic potential is often similar to the use of the conventional potential with extreme damping of the interior and surface contributions. The calculations in this paper were performed to investigate whether the inclusion of coupling to the singlet channel of the neutron–proton system would invalidate any of the results which were obtained earlier without including this coupling.

It may be noted that the equations giving the structure of the singlet contributions to the stripping amplitude are of a very general nature, and are not limited by the validity of the adiabatic approximations. The contributions of the singlet channel may be split into two categories. First there is the direct singlet contribution from the singlet matrix element, which depends on the amount of coupling between the spin channels, and on the ratio of the singlet and triplet neutron–proton potentials. The effect of this contribution is to modify the $J_1 = l_1$ partial wave in the triplet state and hence the balance between the contributions of different total angular momentum radial integrals for a given orbital angular momentum. In this respect the singlet contribution resembles the contribution of vector and tensor spin–orbit forces in the deuteron channel in the conventional calculation. Secondly, the singlet channels contribute, in second order, to the triplet matrix element. This contribution, which depends on the amount of coupling between the spin channels, can be shown to be equivalent to a non-local tensor spin–orbit force (Harvey 1970). In numerical calculations it is found that the direct singlet contribution is far more important than the corrections to the triplet matrix element. In the next section, the results of using the adiabatic theory are presented, including coupling to the singlet channel for a typical reaction.

3. Calculations including singlet coupling

As became clear in the previous section, the difference of the nucleon spin–orbit forces plays a crucial part in the inclusion of singlet S-waves in stripping reaction calculations. Indeed, if these forces are assumed equal (as is generally the case) then the singlet S-waves play no part in stripping reactions. Nucleon spin–orbit forces are at present poorly determined, and there is ample scope for an isobaric spin dependence of the spin–orbit potential, on the available experimental evidence. In the latest determination of the systematic optical potentials (Bechetti and Greenlees 1969), the spin–orbit strength is quoted as $6{\cdot}2 \pm 1$ MeV, leading to values of $|V_n - V_p|$ of up to 2 MeV being not unreasonable. This value is also consistent with results based on bound single-particle properties (Millener and Hodgson 1973). In view of this uncertainty, the approach adopted in the subsequent calculations has been to assume that small differences in the nucleon spin–orbit potentials are consistent with the experimental evidence,

and to investigate the relationship of the results to these small differences to determine the qualitative effects of singlet coupling in stripping reactions.

The coupled equations (32) have been solved in a symmetric approximation for $l_1 = J_1$, under which circumstances they can be decoupled by addition and subtraction, and there are then four equations to solve for a given l_1 value in the deuteron channel. The equations are very nearly symmetric, since the only asymmetry is in the lack of the spin–orbit potential in $\overline{V}^{l_1}_{00}$ compared with $\overline{V}^{l_1 l_1}_{11}$, and in the energy term $(\epsilon_d + \epsilon_0)$. It is assumed throughout the adiabatic calculation that the dependence upon these energies is negligible, and the spin–orbit term is approximately 5% of the strength of the real parts of $\overline{V}^{l_1}_{00}$ and $\overline{V}^{l_1 l_1}_{11}$. The coupling terms are symmetric in zero-range and finite-range approximations and are easily calculated, since they have the same radial dependence as the spin–orbit potentials.

It is convenient to note here, that in first order, the magnitude of singlet radial wavefunctions is proportional to the strength of the coupling term, and hence to the difference of the spin–orbit forces. The convention used for defining the spin functions

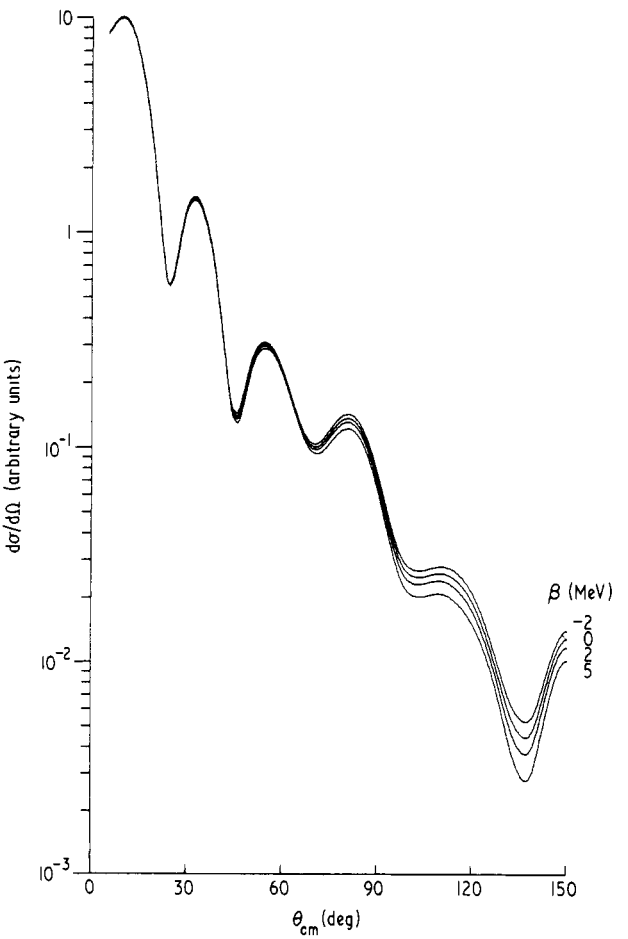

Figure 1. Differential cross sections including singlet coupling effects for the reaction ^{54}Fe(d, p)^{55}Fe at 23 MeV calculated for various values of the isospin dependence of the nucleon spin–orbit potential ($\beta = V_p^{so} - V_n^{so}$).

$\chi_{s_1}^{\sigma_1}$ is relevant here, since changing the labelling of neutron and proton changes the sign of the coupling term, and of $\chi_{s_1=0}^{\sigma_1}$. The difference of the spin–orbit forces $(V_p^{so} - V_n^{so})$ is designated by β in the figures. The fact that the singlet radial wavefunctions are proportional to β can be shown to lead, in first order, to angular distributions and polarizations involving a term linear in β, that is, of the form

$$\sigma(\theta) = \sigma_1(\theta) + \beta\sigma_2(\theta). \tag{47}$$

The results of a series of calculations for the deuteron stripping reaction on ^{54}Fe at 23 MeV are shown in figures 1 to 3. It can be seen that the effects of singlet coupling, like those of spin–orbit forces, are large angle effects, and small. Even the unphysical value of $\beta = 5$ MeV, leads to no dramatic effect on the cross section in figure 1. It should be noted here, that the calculations leading to the curves shown were not completely consistent, inasmuch as the spin–orbit force in the proton channel, and that used to generate the neutron form factor, were not altered while the parameter β was changed. The effects on the deuteron analysing power, and tensor asymmetries, shown in figures 2 and 3 (Madison convention, Barschall and Haeberli 1971) are larger. However, these calculations do not include deuteron D-state effects which are known to be crucial in an understanding of deuteron asymmetries (Brown *et al* 1971, Johnson *et al* 1973, Rohrig and Haeberli 1973, Knutson *et al* 1973). These curves are useful therefore in so far as they indicate the order of magnitude of singlet break-up effects and will be

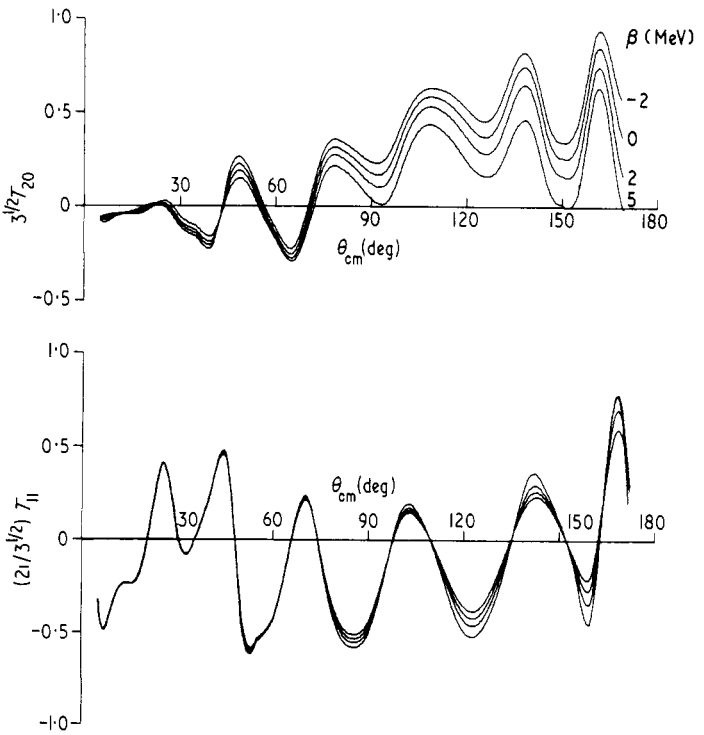

Figure 2. Calculated deuteron analysing power and tensor asymmetry for the reaction ^{54}Fe(d, p)^{55}Fe including singlet coupling effects.

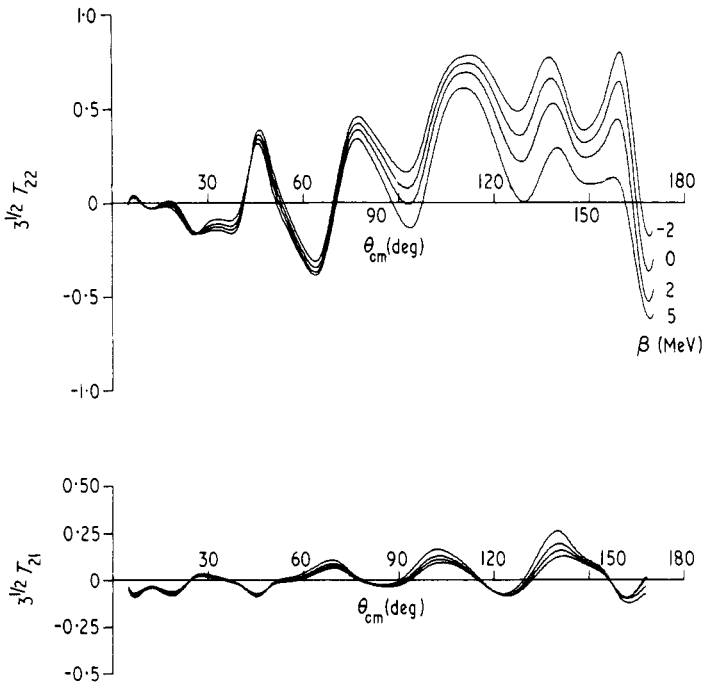

Figure 3. Calculated tensor asymmetries for the reaction ^{54}Fe(d, p)^{55}Fe including singlet coupling effects.

directly relevant to experiment only when a very detailed fit is attempted. It can be seen that the first-order expansion (41) is quite accurate.

The calculations were performed using nucleon potentials from Becchetti and Greenlees (1969) as discussed in Harvey and Johnson (1971). The value 0·7 was used for the ratio α (equation (27)) this being the value obtained from the Yamaguchi (1954) potential when the singlet and triplet ranges are assumed to be equal.

4. Summary

The investigation of the effects of coupling to the singlet neutron–proton break-up states required the use of a reformulated stripping matrix element consisting of the sum of singlet and triplet matrix elements. With the effective potential chosen as the sum of the neutron and proton optical potentials, the singlet contributions vanish in the case where there is no isospin dependence of the spin–orbit potential. For reasonable values of the difference in spin–orbit potentials, it was found that rather small differences were made to the predictions of the simple theory which includes only triplet contributions. The singlet coupling effects were found to be greatest on the tensor polarizations and asymmetries. These results support the neglect of singlet contributions in the investigation of elastic scattering, using the adiabatic theory, in the work of Johnson and Soper (1970) and in the analysis of deuteron stripping reactions using the adiabatic theory, and in any case these effects were found to be smaller than deuteron D-state effects. The essential point is that although the large observed value of the singlet n–p scattering

length suggests the singlet S-wave n–p continuum should play a role comparable to that of the triplet n–p continuum, this consideration is outweighed by the fact that the singlet S channels are coupled to the triplet S channels only through the difference between the neutron and proton spin–orbit forces, and our quantitative results show that for reasonable physical values of this difference the net effect on (d, p) observables is generally small.

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